MATHEMATICAL STUDY OF THE EFFECT OF TUNED LIQUID DAMPERS IN MITIGATING THE EFFECTS OF VIBRATIONS ON BUILDINGS RESULTING FROM EARTHQUAKES AND EXPLOSIONS

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ABSTRACT

Recently, accidents at critical infrastructure facilities due to seismic activity, military explosions and accidents have become more frequent. The importance of studying such events in terms of the preservation and improvement of the environment and the sustainable development of any country is beyond doubt. These man-made effects negatively affect the environment. The hazardous materials released pollute soil, and surface and groundwater. Dust and emissions reduce the quality of the atmosphere. Destroyed objects could disrupt and degrade ecosystems resulting in a reduction in biodiversity. In this regard, a comprehensive analysis of the system to increase the environmental safety level of buildings and different technogenic objects under seismic loads, military explosions and emergencies, the development of theoretical models and practical solutions to ensure their stability and reduce environmental threats, is an urgent scientific task. The aim of this paper is to study the effectiveness of tuned liquid dampers in mitigating structural damage when subjected to vibrations. Tuned liquid dampers are strong tanks, partially filled with liquid, which are strongly integrated into a flexible structure. A numerical model that incorporates the interaction between a structure and a tuned liquid damper was developed. The structure is assumed to be a single degree of freedom system. To obtain the fundamental frequencies of a tuned liquid damper, the boundary element method was used. The novelty of the proposed method is that it defines the mitigation of vibrations in a structure with a tuned liquid damper.

Keywords: critical infrastructure; environmental safety; mitigation of vibrations; seismic loads; tuned liquid damper; vibration control

Introduction

Recently, damage to important facilities due to earthquakes, military explosions and accidents have become more frequent. The relevance of studying the effect of earthquakes on such facilities in order to preserve and improve the environment and ensure sustainable development of a country is beyond doubt.

Buildings and facilities destroyed by earthquakes negatively affect the environment as dangerous materials from these structures pollute the soil and surface and groundwater and dust reduce the quality of the atmosphere. In addition, they could disrupt and degrade ecosystems and adversely affect biodiversity.

Thus, it is important to increase the environmental safety level of buildings and other structures in areas subject to earthquakes and explosions, which can be done by developing theoretical models and practical solutions that ensure their stability and reduce the threat to the environment.

Problem Formulation

The increase in critical incidents has resulted in research on the vibration damping of elastic structures. A large number of damping devices have been developed to reduce vibrations (Ghaedi et al. 2017). Among them are membranes (Choudhary et al. 2021) and baffles (Zhang et al. 2020) in fuel tanks, as well as shock-absorbing suspensions in vehicles (Fang et al. 2024). Tuned liquid dampers (TLDs) are an effective means of suppressing vibrations that are currently widely used as passive or semi-active control devices for controlling vibrations of tall buildings and other high-rise structures with different dynamic loading conditions. These damping devices consist of rigid, thin-walled structures partially filled with liquids that are integrated into a structure. The inclusion of TLDs in structures can enhance their performance, particularly during high winds and earthquakes. The sloshing of the fluid in TLDs can generate pressures that alter both the dynamic characteristics of the structure and its response to vibrations. Tuning the sloshing frequency of the TLD to a structure's natural frequency results in sloshing and breaking waves at the resonant frequencies of the combined TLD-structure. Therefore, the problem of determining the fundamental frequencies of liquid sloshing in rigid tanks is important for research on problems of vibration damping.

The use of TLDs to reduce structural vibration in civil engineering was first proposed by Bauer (1984), who suggested the use of rectangular containers complete-

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ly filled with two immiscible liquids. Since then, many studies have resulted in control devices for stabilizing tall buildings (Wang et al. 2021; Wang et al. 2024), vehicle suspension (Fang et al. 2024), as well as more effective numerical methods for analyzing liquid sloshing (Gnitko et al. 2019; Saghi et al. 2021; Zheng et al. 2021) and structure stability (Zang et al. 2020; Smetankina et al. 2023). The finite element (Zaitsev et al. 2020; Konar and Ghosh 2023) and finite volume (Rusanov 2020) methods are highly efficient for solving problems related to fluid-structure interaction. The use of tuned liquid dampers with baffles is reported in Rusanov et al. (2020) and Konar (2024). The effect of horizontal and vertical baffles on sloshing frequencies in tanks used as liquid dampers, is reviewed by Strelnikova et al. (2020).

TLDs are especially effective in suppressing vibrations of structures during earthquakes (Gnitko et al. 2011), high winds (Liu et al. 2022) and explosions (Sierikova et al. 2023). However, these effects are often characterized by uncertainties. Sloshing in tanks of different forms is reviewed by Sierikova et al. (2022b). In addition, the improving of the Mechanical Properties of Liquid Hydrocarbon Storage Tanks is reported by Sierikova et al. (2022a) and strengthening of the steel of the tanks by chemical-thermal treatment is reported by Savchenko et al. (2022).

Problem Solution

The basic consideration is a single degree of freedom elastic system equipped with TLD, designed to control oscillations induced by external loads. The TLD absorbs energy by the liquid in the tank moving in the opposite direction to the movement in the structure. Fig. 1a illustrates this concept schematically, showing the motion of the liquid relative to the structure.

Equations of crisp boundary value problem for liquid motion

Rigid containers in TLDs are generally rectangular or circular. In this study an arbitrary shell of revolution is

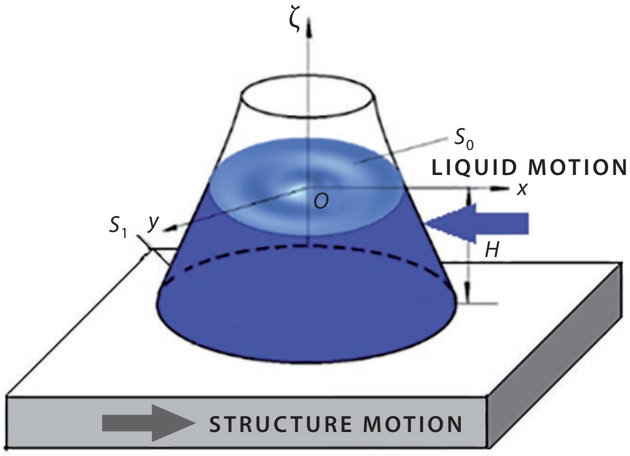

Fig. 1 Structure with tuned liquid damper.

the TLD tank, Fig. 1). The liquid inside the tank is assumed to be incompressible. and are the time-dependent liquid free surface and shell wetted surface. Assuming that initially $S_0(0) = \Sigma_0$ and $S_1(0) = \Sigma_1$ and the free surface Σ_0 is located in the plane z = 0 when the system is in a state of rest.

The domain $Q(t)$, occupied by the liquid, in the cylindrical coordinate system (r, θ, z) is:

$$Q(t) = \{0 \le \theta \le 2\pi, 0 \le r \le r(z), -H \le z \le \zeta(r,\theta,t)\}.$$

In which describes the shell meridian, the unknown function $\zeta = \zeta(\theta, r, t)$ characterizes the time-dependent free surface elevation.

The continuity equation is , where $\mathbf{V} = 0$ is the velocity of the liquid. If the fluid flow is free of vortices, a scalar velocity potential $\Phi = \Phi(x,y,z,t)$ exists, and the continuity equation simplifies to the Laplace equation . When an external force $\mathbf{F}_Q$, with acceleration $\mathbf{a} = a_x(t)\mathbf{i}$ is applied to the liquid-filled shell in the horizontal direction, and the force of gravity in the vertical direction, the equation for liquid movement is derived using the law of momentum conservation as follows:

$$\rho_l\left(\frac{\partial V}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V} + \mathbf{a} + \mathbf{g}\right) = -\nabla p, \quad (1)$$

where p is the liquid pressure, ρ_l is the liquid density, and g is the acceleration due to gravity. Thus, it follows from (1), that for potential flows

$$p - p_0 = -\rho_l\left[\frac{\partial\Phi}{\partial t} + gz + a_x(t)x + \frac{1}{2}(\nabla\Phi,\nabla\Phi)\right]. \quad (2)$$

Here p_0 is atmospheric pressure. Then boundary value problem for the Laplace equation in the time-dependent fluid domain $Q(t)$ is specified. At the wetted surface S_1 the impermeability condition is:

$$\partial\Phi/\partial\mathbf{n}|_{S_1} = 0, \quad (3)$$

where $\mathbf{n}$ is the normal external unit for a wet surface S_1. In addition to the no-slip condition (3) of the wet surface, dynamic and kinematic boundary conditions are imposed on the free surface S_0 as follows:

$$\frac{\partial\Phi}{\partial\mathbf{n}} = \left.\frac{\partial\zeta/\partial t}{\sqrt{1+|\nabla\zeta|^2}}\right|_{S_0},$$

$$\frac{\partial\Phi}{\partial t} + gz + a_x(t)x + \frac{1}{2}(\nabla\Phi,\nabla\Phi) = 0. \quad (4)$$

The solvability condition for Neumann's problem (3)–(4) is derived in Raynovskyy (2020) and takes the form:

$$\iiint_{Q(t)} dQ(t) = 0. \quad (5)$$

The initial data for the boundary value problem (3)–(5) are:

$$\zeta(x,y,0) = \zeta_0(x,y) = 0, \quad \left.\frac{\partial\Phi}{\partial\mathbf{n}}\right|_{\Sigma_0} = \varphi(x,y,0). \quad (6)$$

Thus, the problem of estimating pressure is reduced to determining the unknown functions Φ and ζ using the boundary value conditions (3)–(4) for the Laplace equation, along with initial data (6). To solve this problem, expansions in series using eigenfunctions of the spectral boundary problem (Strelnikova et al. 2020; Raynovskyy and Timokha 2020) were used. Thus, it is necessary to determine the fundamental frequencies and modes of oscillation of the fluid within the rigid shell.

Spectral boundary problem in the shell of revolution

The spectral boundary value problem (Raynovskyy and Timokha 2020) is the determining of the vibration modes and frequencies of liquids in partially filled rigid reservoirs in linear formulation. As the reservoir is assumed to have a rigid shell (Fig. 1b). According to (Raynovskyy and Timokha 2020), the unknown functions ζ i **Φ** in cylindrical coordinates (*r*,q,*z*) are represented by the following:

$$\zeta(r,\theta,t) = \sum_{k=1}^{n} d_k(t)\zeta_k(r,\theta), \quad (7)$$

$$\Phi(r,\theta,z,t) = \sum_{k=1}^{n} \dot{d}_k(t)\varphi_k(r,\theta,z). \quad (8)$$

Here n φ_k(r, θ) and n ζ_k(r, θ) are basic functions, and $d_k(t)$ are unknown time-dependant coefficients. For unknown basic functions in (7)–(8) the linear boundary value problems are (Strelnikova et al. 2020):

$$\nabla^2\varphi_k = 0, \mathbf{P}\in Q(0),\ \frac{\partial\varphi_k}{\partial\mathbf{n}} = 0\Big|_{\Sigma_1}, \frac{\partial\varphi_k(r,\theta,z)}{\partial\mathbf{n}} = \frac{\chi_k^2}{g}\varphi_k(r,\theta,H)\Big|_{\Sigma_0}, \quad (9)$$

where χ_k are the fundamental frequencies.

To solve the boundary problems (9), formulated above, the Green third formula is used as follows:

$$2\pi\varphi(\mathbf{P}_0) = \iint_S \frac{\partial\varphi}{\partial\mathbf{n}}\frac{1}{|\mathbf{P}-\mathbf{P}_0|}dS - \iint_S \varphi\frac{\partial}{\partial\mathbf{n}}\frac{1}{|\mathbf{P}-\mathbf{P}_0|}dS, S = \Sigma_0\cup\Sigma_1 \quad (10)$$

Here, the points **P** and $\mathbf{P}_0$ are located on the integration surface *S*, and $|\mathbf{P} - \mathbf{P}_0|$ are the Cartesian distance between these points and index *k* is omitted in (10) for simplicity. Applying boundary conditions of spectral boundary value problem (9), one obtains the following system of singular integral equations:

$$2\pi\varphi(\mathbf{P}_0) + \iint_{S_1}\varphi\frac{\partial}{\partial\mathbf{n}}\left(\frac{1}{|\mathbf{P}-\mathbf{P}_0|}\right)dS_1 - \frac{\chi^2}{g}\iint_{S_0}\frac{\varphi}{|\mathbf{P}-\mathbf{P}_0|}dS_0$$
$$+ \iint_{S_0}\varphi\frac{\partial}{\partial\mathbf{n}}\left(\frac{1}{|\mathbf{P}-\mathbf{P}_0|}\right)dS_0 = 0,\ \mathbf{P}_0\in S_1, \quad (11)$$

$$2\pi\varphi(\mathbf{P}_0) + \iint_{S_1}\varphi\frac{\partial}{\partial\mathbf{n}}\left(\frac{1}{|\mathbf{P}-\mathbf{P}_0|}\right)dS_1$$
$$+ \left(-\frac{\chi^2}{g}\right)\iint_{S_0}\frac{\varphi}{|\mathbf{P}-\mathbf{P}_0|}dS_0 = 0,\ \mathbf{P}_0\in S_0, \quad (12)$$

In the following, shells of revolution are considered. For this purpose, the series below are employed:

$$\zeta(r,\theta,t) = \sum_{k=1}^{n}\sum_{l=0}^{m} d_{kl}(t)\cos(l\theta)\zeta_k(r),$$
$$(r,\theta,z,t) = \sum_{k=1}^{n}\sum_{l=0}^{m} \dot{d}_{kl}(t)\cos(l\theta)\varphi_k(r,z),$$
$$\frac{\partial\varphi_k}{\partial\mathbf{n}} = \frac{\chi_k^2}{g}\varphi_k\Big|_{\Sigma_0} \quad (13)$$

where *l* is the wave number.

This leads to the next set of one-dimensional singular integral equations:

$$2\pi\varphi(r_0,z_0) + \int_\Gamma \varphi(r(z),z)\Theta(z,z_0)r(z)d\Gamma$$
$$-\frac{\chi^2}{g}\int_0^R \varphi(\rho,H)\Xi(\mathbf{P},\mathbf{P}_0)\rho d\rho = 0.$$
$$2\pi\varphi(r_0,H) + \int_\Gamma \varphi(r(z),z)\Theta(z,z_0)r(z)d\Gamma$$
$$-\frac{\chi^2}{g}\int_0^R \varphi(\rho,H)\,\Xi(\mathbf{P},\mathbf{P}_0)\rho d\rho = 0, \mathbf{P}_0\in S_0. \quad (14)$$

with the following kernels

$$\Theta(z,z_0) = \frac{4}{\sqrt{a+b}}\left\{\frac{1}{2r}\left[\frac{r^2-r_0^2+(z_0-z)^2}{a-b}E_l(k)\right.\right.$$
$$\left.\left.-F_l(k)\right]n_r + +\frac{z_0-z}{a-b}E_l(k)n_z\right\},$$
$$\Xi(P,P_0) = \frac{4}{\sqrt{a+b}}F_l(k), a = r^2 + r_0^2 + (z-z_0)^2,$$
$$b = 2rr_0$$

The generalized elliptic integrals are introduced beforehand as

$$E_l(k) = (-1)^l(1-4l^2)\int_0^{\pi/2}\cos 2l\theta\sqrt{1-k^2\sin^2\theta}\,d\theta$$

$$F_l(k) = \int_0^{\pi/2}\frac{\cos 2l\theta d\theta}{\sqrt{1-k^2\sin^2\theta}}d\theta, k^2 = 2b/(a+b). \quad (15)$$

The numerical implementation of equations (14) was done using the boundary element method with constant approximation of the density (Brebbia et al. 1984), while the generalized elliptic integrals (15) were computed using the method of Karaiev and Strelnikova (2020).

After determining the basic functions and fundamental vibration frequencies, substitute functions cos(*l*θ)$\zeta_k(r)$ and cos(*l*θ)$\varphi_k(r, z)$ into linearized dynamic condition (4) of the free surface and used to derive differential equations for unknowns $d_{kl}(t)$. The acceleration **a** appearing in the dynamic boundary condition represents the acceleration of the elastic body, the vibrations of which need to be reduced using a tuned liquid damper.

Resolving system of differential equations

The coupled problem formulation of the vibration damping of an elastic structure using a tuned liquid damper can be presented in the following form (Gnitko et al. 2011):

$$[\mathbf{M}_S]\{\ddot{\mathbf{u}}^s\} + [\mathbf{C}_S]\{\dot{\mathbf{u}}^s\} + [\mathbf{K}_S]\{\mathbf{u}^s\} = \{\mathbf{f}_S\} + \{\mathbf{f}_{pr}\} \quad (16)$$

where $[\mathbf{M}_s]$, $[\mathbf{C}_s]$, $[\mathbf{K}_s]$, are matrices of masses, damping and stiffness, respectively, $\{\mathbf{f}_s\}$ is an unknown structure displacement vector, is the vector of applied forces acting on the elastic structure, and $\{\mathbf{f}_{pr}\}$ is the force vector representing the fluid pressure from the TLD. To obtain the pressure vector $\{\mathbf{f}_{pr}\}$, the results of the previous section are used. Namely, the linearized dynamic boundary condition on the free surface is used for estimating the unknown coefficients $d_{kl}(t)$. This results in the following differential equations:

$$[\boldsymbol{M}_f]\{\ddot{\mathbf{d}}\} + [\boldsymbol{K}_f]\{\mathbf{d}\} + \rho_l\{\mathbf{a}\} = 0, \tag{17}$$

where $\mathbf{a}$ is the acceleration due the elastic movement of the structure.

Thus, the coupled system of differential equations (16)–(17) can be used to carry out a dynamic analysis of the behaviour of an elastic structure in the presence of TLD in the form

$$[\mathbf{M}_S]\{\ddot{\mathbf{u}}^s\} + [\mathbf{C}_S]\{\dot{\mathbf{u}}^s\} + [\mathbf{K}_S]\{\mathbf{u}^s\} =$$
$$\{\boldsymbol{f}_S\},+\{\mathbf{f}_{pr}\}[\boldsymbol{M}_f]\{\ddot{\mathbf{d}}\} + [\boldsymbol{K}_f]\{\mathbf{d}\} + \rho_l\{\ddot{\mathbf{u}}^s\} = 0. \tag{18}$$

To solve this zero was introduced into the initial conditions, ensuring that the "elastic structure-tuned liquid damper" system is initially at rest.

In the first stage, the problem of determining the frequencies and modes of the elastic structure is solved without considering the tuned liquid damper effect. If damping is not taken into account, the following equations are obtained:

$$[\boldsymbol{M}_S]\{\ddot{\mathbf{u}}^s\} + [\boldsymbol{K}_S]\{\mathbf{u}^s\} = 0. \tag{19}$$

By solving this problem, the natural vibration frequencies Ω_k and modes $\mathbf{u}_k(r, \theta, z)$ are obtained, which here after are the basic functions. Consequently, the displacements of the coupled problem are presented as a series

$$\mathbf{u}^s(x, y, z, t) = \sum_{k=1}^{N} c_k(t)\mathbf{u}_k(x, y, z), \tag{20}$$

where unknown coefficients $c_k(t)$ are treated as generalized coordinates.

Next, the spectral boundary value problem of determining the frequencies and modes of liquid vibrations in the rigid tank are solved (Gnitko et al. 2019; Raynovskyy and Timokha 2020).

With the basic functions $\varphi_k(r, z)$ and $\zeta_k(r)$ known, the vector $\{\mathbf{f}_{pr}\}$ will be obtained applying the pressure p derived from the relations

$$\mathbf{p} = p\,\mathbf{n},\ p = \rho_l\left(\frac{\partial\Phi}{\partial t} + gz\right),\ p = \rho_l[\sum_{k=1}^{n}\ddot{d}_k(t)\varphi_k(r, z)$$
$$+\, gH + \sum_{k=1}^{n} d_k(t)\zeta_k(r)]. \tag{21}$$

The value of $\{\mathbf{f}_{pr}\}$ is obtained by integrating the product of the pressure, given by equation (21), and the eigenmodes $\mathbf{u}_k(x, y, z)$ of the elastic structure over the wetted surface adjacent to the structure.

Accepting $c_{kl}(t)$ and $d_{kl}(t)$ as generalized coordinates enables the reduction of the resolving system of the differential equations for each wave number l to

$$[\mathbf{M}][\ddot{\mathbf{c}}] + [\mathbf{C}][\dot{\mathbf{c}}] + [\mathbf{K}][\mathbf{c}] + [\mathbf{R}][\ddot{\mathbf{d}}] = \{\tilde{\mathbf{f}}_s\}[\mathbf{M}_f]\{\ddot{\mathbf{d}}\}$$
$$+ [\mathbf{K}_f]\{\mathbf{d}\} + \rho_l\{\ddot{\mathbf{c}}\} = 0. \tag{22}$$

Here

$$[\mathbf{M}] = \{\mathbf{M}_S\mathbf{u}_k, \mathbf{u}_j\}, [\mathbf{C}] = \{\boldsymbol{C}_S\mathbf{u}_k, \mathbf{u}_i\}, [\mathbf{K}] = \{\mathbf{K}_S\mathbf{u}_k, \mathbf{u}_j\},$$
$$[\mathbf{R}] = \{\mathbf{f}_{pr}, \mathbf{u}_j\}, \{\tilde{\mathbf{f}}_s\} = \{\mathbf{f}_s, \mathbf{u}_j\}$$

In this study the analysis addresses horizontal external excitation was considered, focusing solely on the first wave component ($l = 1$) in the series expansions of Φ and ζ.

Results and Discussion

Benchmark test

The fundamental frequencies are calculated according to (Afsari et al. 2022) using the reduced boundary element method in linear formulation. The total number of boundary elements along the shell meridians and radii of the free surfaces is 240 for cylindrical shells.

The sloshing frequencies of the liquid in the rigid cylindrical shell with the filling level $H = 1$ m, and radius $R = 1$ m have been obtained.

A comparison of numerical and analytical results from (Raynovskyy and Timokha 2020) for $l = 0$ (axisymmetric modes) for different k is provided in Table 1.

Table 2 provides a comparison of numerical and analytical results (Raynovskyy and Timokha 2020) for $l = 1$ (non-axisymmetric modes) with varying k.

The above results demonstrate the accuracy of the proposed numerical method, exhibiting close agreement with the analytical solutions.

Table 1 Axisymmetric sloshing frequencies, Hz, $l = 0$.

k	Analytical solution	Numerical solution
1	3.828	3.828
2	7.015	7.015
3	10.173	10.176
4	13.323	13.326
5	16.470	16.475

Table 2 Non-axisymmetric sloshing frequencies, Hz, $l = 1$

k	Analytical solution	Numerical solution
1	1.667	1.667
2	5.330	5.330
3	8.536	8.536
4	11.706	11.709
5	14.863	14.866

Analysis of the tuned liquid damping

This study investigates the effect of a liquid damper in reducing the vibration amplitudes of an elastic plate under periodic loading. The analysis is restricted to a one degree of freedom elastic structure represented by the quadratic elastic steel plate, with following parameters: side length of 5 m, thickness of 0.1 m, Young's modulus $E = 210$ GPa, Poisson's ratio $\nu = 0.3$, and the material density $\rho_s = 7900\,\text{kg/m}^3$.

The following function is chosen as loading dependent in time $a_x(t) = a_0 \cos(f_0\, t)$, with different frequencies $f_0 = (2.0, 5.9, 6.6)$, and amplitude $a_0 = (0.01, 0.05, 0.1)$. Using the method of Liu et al. (2022), the modes of free vibrations are obtained as beam functions that describe the displacements of a plate clamped on all edges.

The first frequency of the plate was estimated as

$$\Omega_1 = \frac{\pi^2 h}{a^2}\sqrt{\frac{EF}{6\rho_s(1-\nu^2)}} = 6.643\text{Hz}, \tag{23}$$

where for the first frequency parameter F 22.52 according to Liu (2022). The liquid tuned damper is a rigid cylindrical shell, partially filled with an ideal incompressible liquid. The cylinder radius R is 1m, and the filling level H is 1m. Figs 2–4 show the tuned liquid damper's mitigating effect on the plate vibrations.

In the figures, the navy lines are the displacements in the absence of damper and black lines the displacements considering the influence of damping. These results indicate that, despite detuning from the lower frequency of the plate vibrations, both a beat regime (Fig. 4) and a linear increase (Fig. 5) in vibration amplitude is recorded. It is important to note that the vibrations of the plate were considered without including damping [matrix $\mathbf{C} = 0$ in equations (21)]. However, the installation of the tuned liquid damper significantly reduced the amplitudes of the plate vibrations in all the cases considered (Grigorenko and Efimova 2005; Ahmadiani et al. 2018; Wang et al. 2020; Liu et al. 2022).

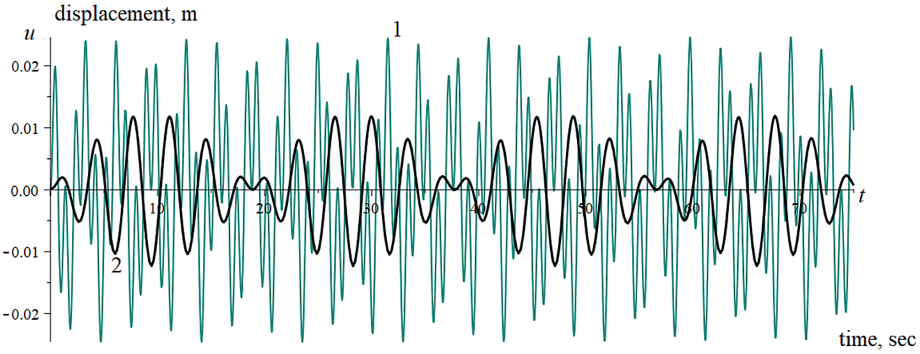

Fig. 2 Plate displacement over time at $f_0 = 2$Hz, $a_0 = 0.01$m.

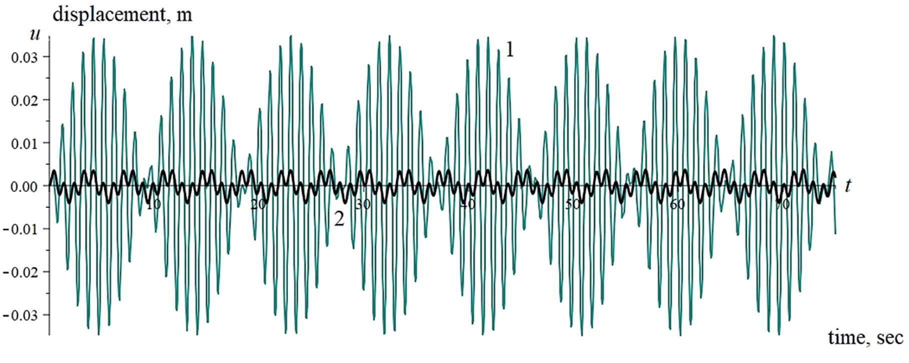

Fig. 3 Plate displacement over time at $f_0 = 5.9$Hz, $a_0 = 0.05$m.

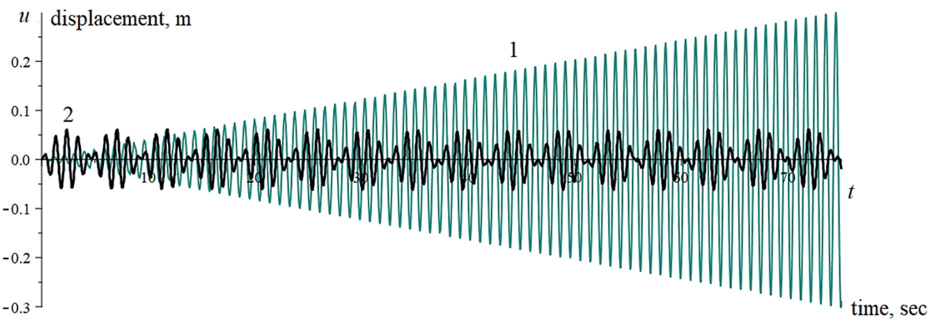

Fig. 4 Plate displacement over time at $f_0 = 6.6$Hz, $a_0 = 0.1$m.

Moreover, even in the case when the forcing frequency nearly coincides with the lowest natural frequency of the plate vibrations and the forcing amplitude is the largest among the studied cases, the tuned liquid damper still provides effective suppression of the plate oscillations.

Thus, the application of a tuned liquid damper consistently results in a substantial decrease in plate vibration amplitudes, in full agreement with the findings of other researchers.

Conclusion and further research

A method is developed for studying the influence of a tuned liquid damper in mitigating the vibration of an elastic structure. The method is based on the use of integral equations to determine the frequencies and vibration modes of a tuned liquid damper. The effective numerical procedure is elaborated to numerical estimation for the singular integrals.

The use of a liquid damper resulted in a significant reduction in the amplitude of vibration of an elastic element over the load range used. The use of liquid dampers will ensure the stability of buildings and critical infrastructure facilities and reduce environmental threats posed by earthquakes and explosions.

In the future work, the focus will be on developing the capability to account for uncertainties in the specification of external loads. Preliminary studies have already examined seismic excitations with different prevailing frequencies. In addition, the influence of other reservoir geometries will be investigated to extend the applicability of the proposed approach.

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